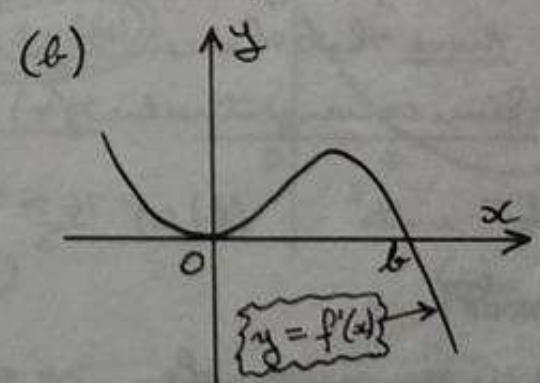
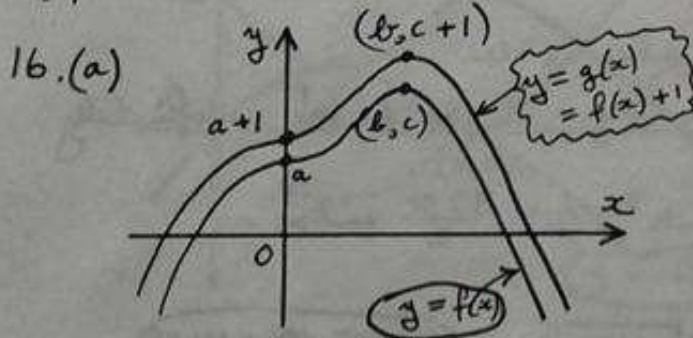


1997 S.C.E. HIGHER I (ANSWERS)

1. $2x + y = -3$ 2. Verify that $\vec{AB} = \begin{pmatrix} -1 \\ -4 \\ 2 \end{pmatrix}$ $\vec{BC} = \begin{pmatrix} 2 \\ -8 \\ 4 \end{pmatrix}$
 $\therefore \vec{AB} = \frac{1}{2} \vec{BC}$
 $\therefore AB$ is parallel to BC
 $\therefore A, B, C$ are collinear (\because of the common point B)
3. $f[g(x)] = 2(\sin x + \cos x)$
 $g[f(x)] = \sin 2x + \cos 2x$
4. (a) $\vec{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$ (b) $PQ = 9$ units
5. (a) a real root is $x = 2$ (b) Other roots come from $2x^2 + x + 4 = 0$
 $\therefore \Delta = -31$

Since $\Delta < 0$, there are no other real roots

6. $x = -1$ 7. [You will need to construct a Δ from $\tan x = \frac{4}{3}$].
8. $\frac{dy}{dx} = 4x + 1$; then replace $\frac{dy}{dx}$ by $4x + 1$ in $x(1 + \frac{dy}{dx})$.
9. (a) Prove that $f(x) = 2(x+2)^2 - 11$ (b) MIN T.P. is $(-2, -11)$
10. $\frac{14}{3}$ 11. $\sqrt{29} \sin(x - 68.2^\circ)$ 12. $(x-13)^2 + (y-4)^2 = 17$
13. $\underline{a} \cdot (\underline{b} + \underline{c}) = 0$; \underline{a} is perpendicular to $\underline{b} + \underline{c}$.
14. $c < 13$ 15. $f(x) = \frac{1}{2} \sin 2x + \frac{3}{4}$



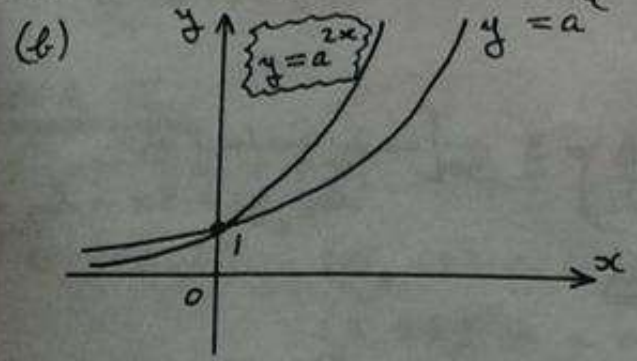
(c) $g'(x)$ is the same graph as $f'(x)$

17. $A(-4\frac{1}{2}, 0)$ $B(14.91, 8)$

18. (a) [Replace $\cos 2x$ by $1 - 2\sin^2 x$ and $\cos^2 x$ by $1 - \sin^2 x$]
 (b) $x = 19.45^\circ, 160.55^\circ, 270^\circ$

19. (a) $t = a$; $u = 0$

(c) Point of intersection is $(1, a^2)$



20. $a = 51.34^\circ$

HIGHER II

(a) $A(1, 3) B(-3, -5)$ (b) (i) $C(-5, 1)$ (ii) $x + 2y = -3$

(a) $R(7, -1, 6)$ (b) $\left[\vec{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix} \vec{SP} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \right] \quad \angle PSR = 84.56^\circ$

3. (a) $\left[\begin{array}{l} u_0(1^{st} \text{ JAN}) = 1000 \quad u_1(1^{st} \text{ FEB}) = 1105 \quad u_2(1^{st} \text{ MAR}) = 1210.53 \\ u_3(1^{st} \text{ APRIL}) = 1316.58 \quad u_4(1^{st} \text{ MAY}) = 1423.16 \quad u_5(1^{st} \text{ JUNE}) = 1530.28 \quad u_6(1^{st} \text{ JULY}) = 1637.93 \end{array} \right]$
 \therefore Amount on JUNE 30th = £1537.93

(b) $\left[\text{Continue until } u_{10}(1^{st} \text{ NOV}) = 2073.94 \right] \therefore$ First exceeds £2000 on 1st NOVEMBER

(c) $u_{n+1} = 1.005 u_n + 100$ where u_n is the amount in account n months after 1st JAN on 1st of month (where $u_0 = 1000$).

4. (a) $P(1, 0) Q(2, 0)$ (b) $\left[\text{TOTAL AREA} = 1\frac{11}{12} + \frac{7}{12} \right] = \underline{2\frac{1}{2} \text{ units}^2}$

5. (a) $A(0, 5) B(2, 1)$ (b) $\left[\text{FIRST PROVE THAT CURVES MEET WHEN } x = 0 \text{ or } 4 \right]$
 \therefore AREA ENCLOSED = $21\frac{1}{3} \text{ units}^2$

(c) $m = 10, n = -1$

6. (a) $m_{TGT} = -\frac{1}{a^2}$ (b) - (c) (i) $\left[\text{PROVE THAT } B \text{ IS } (0, \frac{2}{a}) \text{ AND } C \text{ IS } (2a, 0) \right]$
 \therefore AREA OF $\triangle OBC = \underline{2 \text{ units}^2}$

(ii) Area of $\triangle OBC$ is always 2 units^2 regardless of position of tangent at A

7. $\frac{5x+1}{(x-4)(x+3)} = \frac{3}{x-4} + \frac{2}{x+3}$

8. (a) $k = -0.000122$ (b) 88.5%

9. (a) LIKELIEST : $h = 3 \quad \rho = 1 \quad \nu = -140^\circ \quad u = 230^\circ$ (b) $\alpha = -0.928$
 OTHERS : $h = 3 \quad \rho = 1 \quad \nu = 220^\circ \quad u = 230^\circ$
 $h = -3 \quad \rho = 1 \quad \nu = 40^\circ \quad u = 230^\circ$
 $h = -3 \quad \rho = 1 \quad \nu = -320^\circ \quad u = 230^\circ$
 $t = 120.53^\circ$

$\left[\text{USE EQUATION WITH REPLACEMENTS FOR } h, \nu \text{ AND } \rho \right]$

10. (a) (i) $\left[\text{USE SIMILAR } \triangle \text{'s } ADE \text{ AND } ABE \right]$ (ii) $\left[V = (2x)(2x)(10 - \frac{5}{2}x) \text{ etc} \right]$

(b) $\left[\text{PROVE THAT GREATEST VOLUME OCCURS WHEN } x = \frac{8}{3} \text{ (not } x = 0 \text{ ; } x > 0) \right]$

\therefore DIMENSIONS ARE $\frac{16}{3} \text{ cm} \times \frac{16}{3} \text{ cm} \times \frac{10}{3} \text{ cm}$

11. (a) (i) - (ii) $OE = 1 + 2 \cos h$ $\left[\text{and finally use } d^2 = OB^2 + OE^2 \right]$

(b) (i) $d^2 = 6 + 4\sqrt{2} \cos(h - \frac{\pi}{4})$ (ii) MAX. VALUE OF d^2 IS $6 + 4\sqrt{2}$; occurs when $h = \frac{\pi}{4}$.

(c) (i) $\left[\text{REPLACE } h \text{ by } \frac{\pi}{4} \text{ in } OB = 1 + 2 \sin h \right]$; $BD = \frac{1}{2}(\sqrt{2} + 2)$

(ii) $\sqrt{6 + 4\sqrt{2}} = 2 + \sqrt{2}$