S.C.E. HIGHER ANSWERS – 1996

PAPER I

1. $3x + 2y = 17$ 2. $a = 2\frac{1}{2}$ 3. $\left(\frac{5\pi}{6}, 3\right)$

4. $2x + y = 10$ 5. 9

6. Verify that $\overrightarrow{AB} = \begin{pmatrix}4\\2\\-2\end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix}6\\3\\-3\end{pmatrix}$ 7. $x(x-1)(x^2+x+1)$

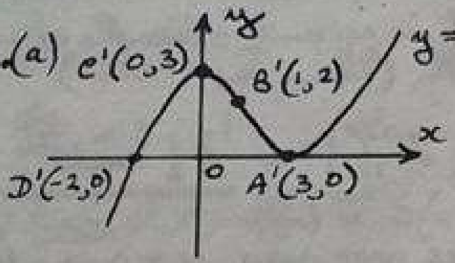
$\therefore \overrightarrow{AB} = \frac{2}{3}\overrightarrow{BC}$

$\therefore$ AB is parallel to BC

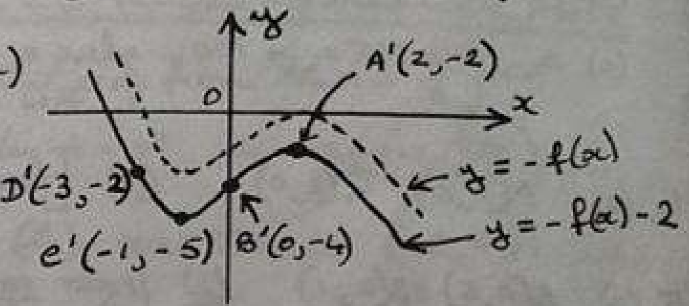
$\therefore$ A, B, C are collinear [$\because$ B is a common point]

B divides AC in the ratio 2 : 3 (ie AB : BC = 2 : 3)

8.(a)



(b)



9. $f'(4) = \frac{5}{16}$

10. $0°, 120°, 180°, 240°$ [NB NOT $360°$ since $0° \leq x < 360°$]

11. (a) Limit exists because 0.3 lies between -1 and 1 (b) $7\frac{1}{7}$

12. $P\left(\frac{\pi}{6}, \frac{1}{2}\right)$ $Q\left(\frac{\pi}{2}, \frac{1}{2}\right)$ 13. $\frac{dy}{dx} = \frac{-\sin x}{2\sqrt{1+\cos x}}$

14. Take 2 lines $\left\{\begin{matrix}2x + 3y = 4\\3x - y = 17\end{matrix}\right\}$. Verify that they intersect at $(5, -2)$.
Then verify that $(5, -2)$ does NOT lie on the 3rd line $x - 3y - 10 = 0$
$\therefore$ 3 LINES ARE NOT CONCURRENT.

15. [Verify that $BD = 5$ and $AD = 2\sqrt{6}$; then use the expansion of $\cos(x+y)$]

16. f is increasing when $x < -2$ AND $x > 3$ [TABLE OF VALUES REQUIRED]

17. $4(x+1)^2 - 9$ 18. $\sin x = \frac{4\sqrt{5}}{9}$

19.(a) $k = 0.019$ (b) [FALLS TO $56.56°C$ IN NEXT 15 MINS]
$\therefore$ IT FALLS $18.44°C$ IN NEXT 15 MINS

20. $(x-3)^2 + (y-4)^2 = 25$.

PAPER II

1.(a) S.P.'s are $(0, 3)$ AND $(3, -24)$ (b) $(0, 3)$ is a point of inflection
$(3, -24)$ is a MINIMUM S.P.

2.(a) Verify that $m_{AB} m_{BC} = -1$ WITH A CLEAR CONCLUSION $\begin{pmatrix}m_{AB} = 2\\m_{BC} = -\frac{1}{2}\end{pmatrix}$

(b)(i) AD: $x - 3y = 6$
BE: $4x + 3y = -1$ (ii) $M\left(1, -\frac{5}{3}\right)$

(a) $Q(2, 2, 9)$ $R(21, 3, 12)$ (b) $Q\hat{P}R = 83\cdot 4^\circ$

4\. (a) (i) $g[f(x)] = 4x^2 + 4x + 1 + k$ (ii) $f[g(x)] = 2x^2 + 2k + 1$

(b) (i) [ANSWER IN QUESTION] (ii) $[\Delta = 64]$ $\therefore$ Roots are REAL AND DISTINCT

(iii) $k = -2$

5\. $\left[\text{area I} = \frac{1}{2} - \frac{\sqrt{3}}{4} \ ; \ \text{area II} = \frac{1}{2}\right]$ $\therefore$ TOTAL AREA $= 1 - \frac{\sqrt{3}}{4}$ units^2

6\. (a) Line l : $y = 2x$ Parabola p : $y = 2x^2$ Circle c : $x^2 + y^2 = 5$

(b) l' : $y = 2x - 4$ p' : $y = -2x^2$ c' : $(x-2)^2 + y^2 = 5$

(c) $(-2, -8)$ AND $(1, -2)$

7\. (a) $\sqrt{13}\cos(x - 56\cdot 3^\circ)$ (b) $138\cdot 3^\circ$, $334\cdot 3^\circ$

8\. (a) $5x + y = -3$; $B(-1, 2)$

(b) $\frac{4}{3}$ units^2

9\. (a) $Y = 3X + 0\cdot 7$

(b) [FIRST REPLACE Y BY $\log_e y$ AND X BY $\log_e x$ TO OBTAIN :- $y = 2\cdot 014x^3$.] $\therefore \begin{Bmatrix} a = 2\cdot 014 \\ b = 3 \end{Bmatrix}$

10\. (a) By symmetry, result needs only to be proved for one point at $(3, 7)$: PROVE THAT TANGENT TO PARABOLA HAS GRADIENT $\frac{2}{3}$ AND PROVE THAT TANGENT TO CIRCLE HAS GRADIENT $-\frac{3}{2}$.

Then conclude that since product of gradients of tangents is -1, the tangents are perpendicular. $\therefore$ CURVES ARE ORTHOGONAL.

(b) (i) $2y = 3x + 23$

(ii) $x^2 + (y - 11\frac{1}{2})^2 = 29\frac{1}{4}$

11\. (a) (i) $h = 5 - x - \frac{1}{2}\pi x$

(ii) [START WITH $L = 2(\text{area of rectangle}) + 1(\text{area of } \tfrac{1}{2} \text{ circle})$]

(b) $x = \frac{20}{8 + 3\pi}$ $(\doteqdot 1\cdot 1478)$

$h = \frac{20 + 5\pi}{8 + 3\pi}$ $(\doteqdot 2\cdot 0493)$

NB : TABLE OF VALUES IS REQUIRED