Higher Maths
SQA 2015 Paper 2
Question 3

A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.

Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.

The toad climbs 13 feet each day before resting.

Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3} f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4} t_n + 13, \qquad t_1 = 13$

where f_n and t_n are the heights reached by the frog and the toad at the end of the nth day after falling in.

(a) Calculate t_2, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

Answers:

(a) $22\frac{3}{4}$ or $\frac{91}{4}$ or $22 \cdot 75$

(b) Calculate the limts for both sequences: 48 for the frog and 52 for the toad.
So only the toad will escape.