Higher Maths: Graph Transformations

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Study this note

We can obtain the graph of

$$y = \mathrm{a}\,f(\mathrm{b}x + \mathrm{c}) + \mathrm{d}$$

by starting with the graph of $y = f(x)$ and transforming each point as follows.

Horizontally, subtract c from each x-coordinate and then divide by b. So $x \mapsto (x - \mathrm{c})/\mathrm{b}$.

This moves each point left c (or right if c < 0). It then either shrinks horizontally (if |b| > 1) or stretches horizontally (if 0 < |b| < 1). If b < 0, there is also a horizontal reflection, in the y-axis.

Vertically, multiply each y-coordinate by a and then add d. So $y \mapsto \mathrm{a}y + \mathrm{d}$.

This either stretches vertically (if |a| > 1) or shrinks vertically (if 0 < |a| < 1). If a < 0, there is also a vertical reflection, in the x-axis. The point then moves up d (or down if d < 0).

In the questions below, the graph of $y = f(x)$ has two turning points, at (2, –5) and (6, 3).

State the coordinates of the related turning points on each of the following graphs.

Q1 Vertical transformations:

a) $y = f(x) + 2$ **b)** $y = f(x) - 1$ **c)** $y = 3f(x)$

d) $y = \frac{1}{2}f(x)$ **e)** $y = -2f(x)$ **f)** $y = -\frac{2}{3}f(x)$

g) $y = 2f(x) + 1$ **h)** $y = -4f(x) - 5$ **i)** $y = 6 - f(x)$

Q2 Horizontal transformations:

a) $y = f(2x)$ **b)** $y = f(-3x)$ **c)** $y = f(x + 2)$

d) $y = f(x - 1)$ **e)** $y = f(2x + 1)$ **f)** $y = f(\frac{1}{2}x - 2)$

g) $y = f(-x + 4)$ **h)** $y = f(-2x + 3)$ **i)** $y = f(-\frac{4}{3}x - 1)$

Q3 Two-dimensional transformations:

a) $y = 2f(x - 3)$ **b)** $y = f(2x) + 4$ **c)** $y = 3f(4x)$

d) $y = -\frac{1}{2}f(-2x)$ **e)** $y = 2 - f(x + 4)$ **f)** $y = f(\frac{1}{3}x + 5) + 4$

g) $y = -2f(x + 1) - 3$ **h)** $y = 3f(\frac{1}{2}x) - 1$ **i)** $y = 3f(7 - x)$

j) $y = 2f(\frac{1}{2}x + 1) - 5$ **k)** $y = 3f(2x - 4) + 1$ **l)** $y = -2f(-\frac{3}{4}x + 2) - 2$