ZETA MATHS

Higher

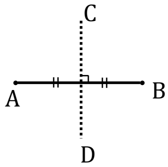
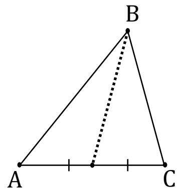
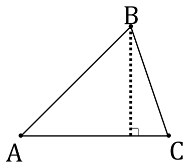
MATHEMATICS

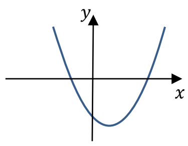
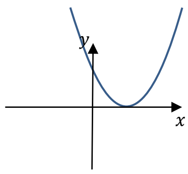
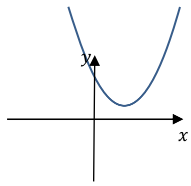
Learning Checklist

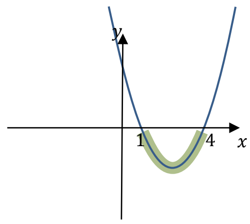
This checklist covers every skill that learners need for success at Higher Mathematics. Each section of this checklist corresponds to the **Zeta Maths Higher Mathematics** textbook (available from **www.zetamaths.com** or **Amazon**). The topic names in this document are linked for easy navigation and colour coded to correspond with skills: **algebraic**, **geometric**, **trigonometric** and **calculus**.

Contents

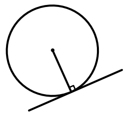
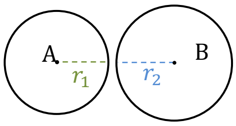
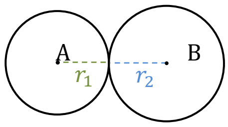
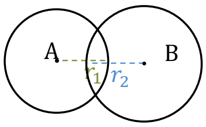
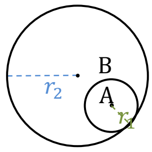
Section	Topic	Skills			
1	**The Straight Line**				
1	The Straight Line	To find the equation of any line, the most important thing to keep in mind is that two pieces of information are needed: **a point on the line** and the **gradient** of the line. This information can be substituted into the gradient point form of the equation of a line: $\boldsymbol{y - b = m(x - a)}$			
National 5 Skills					
1.1	The equation of a line from two points	**Example:** Find the equation of the line joining the points $(4, 1)$ and $(11, -20)$. **Points** $\underset{a\ b}{(\overset{x_1}{4}, \overset{y_1}{1})}\ (\overset{x_2}{11}, \overset{y_2}{-20})$ **Gradient** $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{-20 - 1}{11 - 4}$ $m = \frac{-21}{7}$ $m = -3$ **Equation** $y - b = m(x - a)$ $y - 1 = -3(x - 4)$ $y - 1 = -3x + 12$ $y = -3x + 13$			
1.6	The equation of a line from a point and a gradient	**Example:** Find the equation of the line which passes through $(3, 2)$ and is parallel to the line with equation $3y + 2x = 3$. **Point** $\underset{a\ b}{(3, 2)}$ **Gradient** $3y + 2x = 3$ $3y = -2x + 3$ $y = -\frac{2}{3}x + 1$ $m = -\frac{2}{3}$ **Equation** $y - b = m(x - a)$ $y - 2 = -\frac{2}{3}(x - 3)$ $y - 2 = -\frac{2}{3}x + 2$ $y = -\frac{2}{3}x - 4$			
1.7	Finding the point of intersection of lines	To find the point of intersection of a pair of lines, we use **simultaneous equations**. For simultaneous equations involving two unknowns, there are two common methods: solving by **substitution** or by **elimination**. Often the easiest method to use is **substitution**			
Higher Skills					
1.2	The midpoint of a line segment	• Add the x coordinates of the end points together. • Divide each of them by two. $\textbf{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$			
1.3	The gradients of perpendicular lines	**Perpendicular Lines** are lines that intersect at 90°. When lines are perpendicular, the product of their gradients is equal to -1. We write this as $\boldsymbol{m_1 \times m_2 = -1}$. To calculate a perpendicular gradient, swap the numerator and the denominator and change the sign.			
1.4	The gradient of a line using $m = \tan\theta$	The angle that the line makes with **the positive direction of the x-axis**, namely θ, can be used to calculate the gradient. To do this we use the property: $\boldsymbol{m = \tan\theta}$			

Section	Topic	Skills			
1.5	Collinearity	The term **collinear** describes points or coordinates that are in line with one another, **i.e.** they lie on the same line. To show points are collinear, calculate the gradient of any two line segments joining two pairs of points. These segments must have the same gradient and one of the points must be common to both gradient calculations.			
1.8	Perpendicular bisectors	A **perpendicular bisector** of a line segment is a line that cuts the line segment **in half** (bisects) at **right-angles** (perpendicularly). In the diagram opposite, CD is a perpendicular bisector of AB.  The **gradient** is normally calculated by finding the gradient of AB and then calculating the perpendicular gradient. The **point** is the midpoint of AB.			
1.9	Medians	A **median** in a triangle is a line segment that connects one vertex to the midpoint of the opposite side.  The **point** can be taken from the given vertex and the **gradient** is calculated by finding the midpoint of the opposite side, then using it with the given vertex to calculate the gradient.			
1.10	Altitdues	An **altitude** in a triangle is a line segment from one vertex perpendicular to the opposite side.  The **point** can be taken from the given vertex and the **gradient** is calculated by finding the gradient of the opposite side, then using the property of perpendicular gradients.			
Common Terms					
	Collinear	Three or more points that lie on the same line.			
	Concurrent	Three or more lines that all intersect at the same point. **NB:** In a triangle, altitudes are concurrent (intersect at *orthocentre*), medians are concurrent (intersect at *centroid*) and perpendicular bisectors are concurrent (intersect at *circumcentre*).			
	Centroid	The point of intersection of the three medians of a triangle.			
	Circumcentre	The point of intersection of three perpendicular bisectors in a triangle.			
	Orthocentre	The point of intersection of three altitudes of a triangle.			
2	**Quadratic Functions & Graphs**				
2	Quadratic functions & graphs	A quadratic is a function or equation in which the highest power of x is 2. Quadratic functions are typically written in three forms: **Expanded Form:** $\boldsymbol{y = ax^2 + bx + c}$. **Completed Square Form:** $\boldsymbol{y = k(x+p)^2 + q}$. In this form the turning point is $(-p, q)$. **Factorised Form:** $\boldsymbol{y = k(x-m)(x-n)}$. In this form the roots are $x = m$ and $x = n$.			

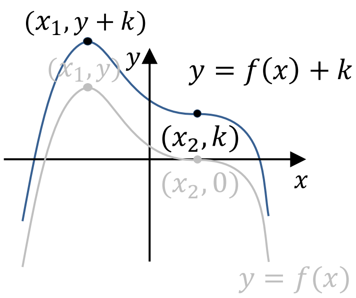
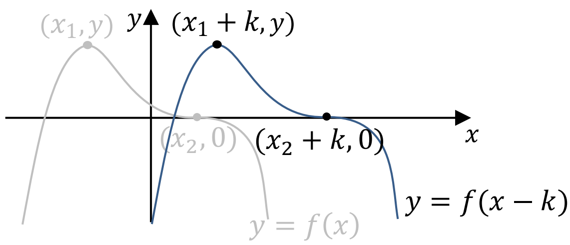
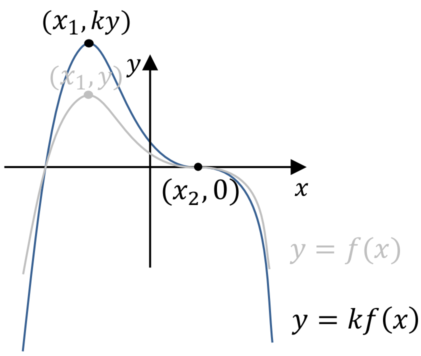
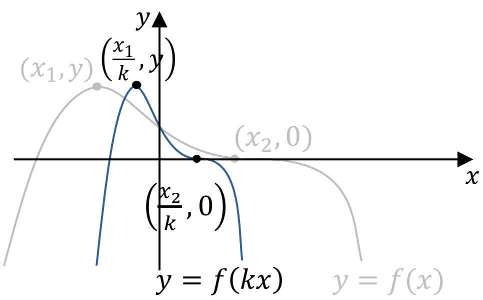
Section	Topic	Skills			
National 5 Skills					
2.1-2	Finding the equation of a quadratic function from a turning point	• For completed square form use $y = k(x+p)^2 + q$ and for root form use $y = k(x-m)(x-n)$. • Substitute the turning point if in completed square form, or the roots, if in root form. • Substitute the other point into the equation to find the value of k.			
2.3-4	Solving quadratics	Graphically, Algebraically or using Quadratic Formula			
2.8-9	Sketching quadratics	• In completed square form. • In root form.			
2.11	Determining the nature of the roots (the discriminant)	The formula $b^2 - 4ac$, where $ax^2 + bx + c = 0$ is known as the **discriminant**. The discriminant is used to determine the *nature of the roots*, or the points of intersection between a quadratic curve and the x-axis. There are three results that matter when calculating the discriminant:  $b^2 - 4ac > 0$ **two real and distinct roots i.e.** the curve cuts the x-axis in two different places.  $b^2 - 4ac = 0$ **two real and equal roots i.e.** the x-axis is a tangent to the curve.  $b^2 - 4ac = 0$ **no real roots** **i.e.** the curve does not cut the x-axis.			
Higher Skills					
2.4	Solviang quadratics by rearranging	If a quadratic equation is not equal to zero, it needs to be rearranged to be solved. **Example:** Solve $x^2 = 4x + 12$. $x^2 - 4x - 12 = 0$ $(x-6)(x+2) = 0$ $x - 6 = 0 \quad or \quad x + 2 = 0$ $x = 6 \qquad\qquad x = -2$			
2.6	Completing the square	In Higher Mathematics we need to complete the square of quadratics with a non-unitary coefficient of x^2, **e.g.** $2x^2, 5x^2$, etc. **Example:** Complete the square of $5x^2 - 5x - 5$. $5x^2 - 5x - 5$ $= (5x^2 - 5x) - 5$ $= 5(x^2 - x) - 5$ $= 5\left(x - \frac{1}{2}\right)^2 - 5 - 5\left(-\frac{1}{2}\right)^2$ $= 5\left(x - \frac{1}{2}\right)^2 - 5 - \frac{5}{4}$ $= 5\left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$			

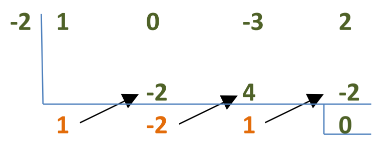
Section	Topic	Skills			
2.7	Solving quadratics by completing the square	**Example:** Solve $x^2 - 6x + 7 = 0$ by first completing the square. $x^2 - 6x + 7 = 0$ $(x^2 - 6x) + 7 = 0$ $(x-3)^2 + 7 - 9 = 0$ $(x-3)^2 - 2 = 0$ $(x-3)^2 = 2$ $x - 3 = \pm\sqrt{2}$ $x = 3 + \sqrt{2}$ *or* $x = 3 - \sqrt{2}$			
2.10	Solving quadratic inequations	To solve a quadratic inequation: • Find the roots. • Draw a sketch. • Answer the question. **Example:** Solve $x^2 - 5x + 4 < 0$. **Roots:** $(x-1)(x-4) = 0$ $x - 1 = 0$ *or* $x - 4 = 0$ $x = 1$ $x = 4$ $\therefore x^2 - 5x + 4 < 0$ for $1 < x < 4$ 			
2.12	Using the discriminant	State the values of k for which the equation $3x^2 + kx + 3 = 0$ has equal roots. Since $ax^2 + bx + c = 0$, then $a = 3, b = k, c = 3$. The quadratic has equal roots so, $b^2 - 4ac = 0$ $k^2 - 4(3)(3) = 0$ $k^2 - 36 = 0$ $k^2 = 36$ $k = \pm 6$			
2.13	Show a line is a tangent to a curve	To show that a line is a tangent to a curve: • Equate the line and the curve. • Rearrange so that they equal zero. • Solve and show equal roots. **Example:** Show that the line with equation $y = 2x + 1$ is a tangent to the curve $y = x^2 + 4x + 2$. $x^2 + 4x + 2 = 2x + 1$ $x^2 + 2x + 1 = 0$ $(x+1)(x+1) = 0$ $x = -1$ twice, $\therefore$ the line is a tangent to the curve.			
2.14	Nature of intersection of a line and a curve	To determine the nature of the intersection of a line and a curve: • Equate the line and the curve. • Rearrange so that they equal zero. • Calculate the value of the discriminant.			
2.15	Points of intersection of a line and a curve	To find the coordinates of the point(s) of intersection of a line and a curve:			

Section	Topic	Skills			
		• Equate the line and the curve. • Rearrange so that they equal zero. • Solve for x, substitute into line to find y.			
2.16	Points of intersection of two curves	To find the coordinates of the point(s) of intersection of a line and a curve: • Equate the line and the curve. • Rearrange so that they equal zero. • Solve for x, substitute into one of the curves to find y.			
3	**The Circle**				
3	The circle	To find the equation of any circle, the most important thing to keep in mind is that two pieces of information are needed: **the centre of the circle** and the **radius** of the circle. This information can be substituted into the Centre-Radius form of the equation: $(x-a)^2+(y-b)^2=r^2$			
National 5 Skills					
3.1	The distance between two points	To find the distance between two coordinates, Pythagoras' Theorem is used. This often appears in the form of the Distance Formula: $Distance=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ It may be simpler and often more helpful to sketch a right-angled triangle and work out the vertical and horizontal difference.			
Higher Skills					
3.2	The equation of a circle – centre, (a, b) and radius, r	**Example:** Find the equation of the circle with centre $(5,-3)$ and radius 5 units. **Centre**: $(5,-3)$, a b **Radius**: $r=5$, $r^2=25$ **Equation**: $(x-a)^2+(y-b)^2=r^2$ $(x-5)^2+(y-(-3))^2=5^2$ $(x-5)^2+(y+3)^2=25$			
3.3	Finding the centre and radius from a circle equation	The centre and radius of the equation of a circle can be derived from both forms of the equation. **Standard (Centre-Radius) Form:** $(x-a)^2+(y-b)^2=r^2$ Centre: (a,b), Radius $=r$ **General (Expanded) Form:** $x^2+y^2+2gx+2fy+c=0$ Centre: $(-g,-f)$, Radius $=\sqrt{g^2+f^2-c}$			
3.4	Coordinates and the circle 1 – within, on, or outside a circle	To determine where a coordinate lies, in relation to a circle, substitute the coordinate into the left-hand side of the equation of a circle. If the answer is **less than the right-hand side** of the equation, the coordinate lies **inside** the circle, if it is **equal to the right-hand side**, the point lies **on** the circle, if it is **greater than the right-hand side**, the point lies **outside** the circle.			

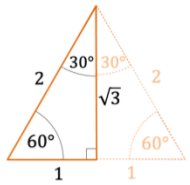
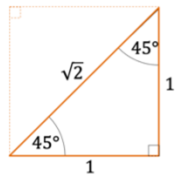
Section	Topic	Skills			
3.5	The nature of the intersection of a line and a circle	To determine the nature of the intersection of a line and a circle: • Substitute the equation of the line into the equation of the circle. • Use the discriminant.			
3.6	The poinst of intersection of a line and a circle	To find the coordinates of the point(s) of intersection of a line and a circle: • Substitute the line into the circle. • Rearrange so that they equal zero. • Solve for x, substitute into the line or circle to find y.			
3.7	The equation of a tangent to a circle	• Determine the gradient of the radius from the centre and the point of contact of the tangent. • Find the of the tangent using perpendicular gradients. • Substitute the perpendicular gradient and the point of contact into the equation of a line, $\boldsymbol{y - b = m(x - a)}$. 			
3.8	The intersection of two circles	 $r_1 + r_2 < \mathrm{AB}$ ∴ circles do not intersect  $r_1 + r_2 = \mathrm{AB}$ ∴ circles touch externally  $r_1 + r_2 > \mathrm{AB}$ ∴ two points of intersection  $r_2 - r_1 = \mathrm{AB}$ ∴ circles touch internally			
3.9	Coordinates and the circle 2 – finding centres and radii	Many circle problems involve using the coordinates of the centre and the radius of a circle to work out the equations of other circles.			
Common Terms					
	Concentric	Circles that share the same centre.			
	Concurrent	Three or more lines that intersect at the same point.			
	Congruent	Any two shapes that have all properties identical, **i.e.** same shape and size.			
4	**Functions**				
4.1	Domain and range of functions	**Domain** The numbers input into a function, $f(x)$, are known as the **domain** of the function. In some functions there are certain values that cannot be input into the function, these are called **restrictions on the domain** of the function. For example, in the function $f(x) = \frac{12x}{(4-x)^2}$ where $x \in \boldsymbol{R}$, $x \neq 4$ as this would make the denominator zero.			

Section	Topic	Skills			
		Range The range of a function is the range of output values. Some functions range from $-\infty$ to $+\infty$, but others have a limited range. For example, the function $f(x) = 3x^2 + 4$ has a minimum turning point at $(0, 4)$, so lowest value of $f(x) = 4$. Therefore, the range of the function is $f(x) \geq 4$.			
4.3	Composite functions	Composite Functions consist of more than one function. Substitution is used to determine the resulting composite function when two functions are given. **Example 1:** If $f(x) = 3x - 2$ and $g(x) = x^2 - 4$, find $f(g(x))$ $f\left(g(x)\right) = 3(x^2 - 4) - 2 = 3x^2 - 12 - 2 = 3x^2 - 14$ **Example 2:** Find $H(-1)$ where $H(x) = g\left(f(x)\right)$, $f(x) = 3x - 2$ and $g(x) = x^2 - 4$. $H(x) = g\left(f(x)\right) = (3x - 2)^2 - 4$ $= 9x^2 - 12x + 4 - 4 = 9x^2 - 12x$ $H(-1) = 9(-1)^2 - 12(-1) = 21$			
4.4	Inverse functions	An Inverse Function is a function that reverses another function, **i.e.** it returns the values in the range of a function to the values of the domain. The notation used for the inverse function is $f^{-1}(x)$. To find an inverse function: • Replace $f(x)$ with y. • Change the subject to x. • Replace y with x and x with $f^{-1}(x)$.			
Common Terms					
	Common notation	$\in$ ‘is an element of’, **e.g.** $3 \in \{0, 1, 2, 3, 4\}$, **i.e.** 3 is an element of the set containing $1, 2, 3$ and 4. $\notin$ ‘is not an element of’, **e.g.** $7 \notin \{0, 1, 2, 3, 4\}$, **i.e.** 7 is not an element of the set containing $1, 2, 3$ and 4.			
	Common Number Sets	There are five standard number sets to consider at Higher Mathematics: **Natural numbers:** $\mathbb{N} = \{1, 2, 3, 4 \ldots\}$ These are the numbers we use for counting. **Whole numbers:** $\mathbb{W} = \{0,1, 2, 3, 4 \ldots\}$ The same as natural numbers, but also including 0. **Integers:** $\mathbb{Z} = \{\ldots, -3, -2, 1, 0, 1, 2, 3, \ldots\}$ The set of positive and negative whole numbers. **Rational numbers:** $\mathbb{Q}$ These are all numbers that can be expressed as a division of two integers. **Real numbers:** $\mathbb{R}$ These are all the real numbers, including irrational numbers (**e.g.** $\sqrt{2}, \pi$, etc.).			

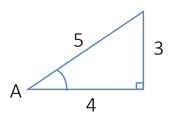
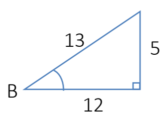
Section	Topic	Skills			
5	**Graphs of Related Functions**				
5	Graphs of Related Functions	When sketching related graphs: • Sketch the original graph lightly on the axes. • Calculate each new coordinate. • Sketch the transformed graph.			
5.1	Graphs of $f(x) \pm k$				
5.2	Graphs of $f(x \pm k)$				
5.3	Graphs of $kf(x)$				
5.4	Graphs of $f(kx)$				

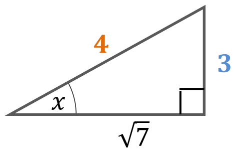
Section	Topic	Skills			
6	**Recurrence Relations**				
6.1	Using recurrence relations	A sequence is defined by the recurrence relation $u_{n+1} = 2u_n - 4$, $u_0 = 24$. Find the value of u_4. $u_0 = 24$ $u_1 = 2u_0 - 4 = 2(24) - 4 = \mathbf{44}$ $u_2 = 2u_1 - 4 = 2(\mathbf{44}) - 4 = \mathbf{84}$ $u_3 = 2u_2 - 4 = 2(\mathbf{84}) - 4 = \mathbf{164}$ $u_4 = 2u_3 - 4 = 2(\mathbf{164}) - 4 = \mathbf{324}$ **NB:** Set up calculator by inputting u_0 and pressing '=' then input $ANS \times 2 - 4$ or $ANS \times a - b$ and continue pressing '=' until the desired answer is reached. Write down all answers.			
6.2	Finding constant values in a recurrence relation	If the constant values $\boldsymbol{a}$ and $\boldsymbol{b}$ of a recurrence relation are unknown, they can be calculated – provided we know three consecutive terms of the sequence. • Substitute the known values into the relation. • Solve simultaneously to calculate the unknown values.			
6.3	The limit of a recurrence relation	When a recurrence relation tends towards a particular value, we say the recurrence relation is **converging**. The value that the recurrence relation tends towards is called the **limit**. This happens when $-1 < \boldsymbol{a} < 1$.			
6.4	Finding the limit of a recurrence relation	The **limit** of a recurrence relation $u_{n+1} = au_n + b$ as $n \to \infty$ can be found using the formula: $\boldsymbol{L = \frac{b}{1-a}}$			
7	**Polynomials**				
7.1	Factorising Polynomials	Use synthetic division. **Example:** Fully factorise $f(x) = x^3 - 3x + 2$. Set up synthetic division using coefficients from polynomial. • If there is no term, use 0. • The value outside the division is derived from factors of the last term (in this case factors of 2). • If the remainder of the division is 0 then the value outside the division is a root.  If $x = -2$ is a root then $(x+2)$ is a factor. $\therefore f(x) = x^3 - 3x + 2 = (x+2)(x^2 - 2x + 1)$ $= (x+2)(x-1)(x-1)$			
7.2	Solving polynomial equations	To solve polynomials, they need to be factorised using synthetic division or algebraic long division and then solved in the same way as quadratic functions. **Example:** Solve $x^3 - 3x + 2 = 0$. Using synthetic division as in 7.1 we have $(x+2)(x-1)(x-1) = 0$ $\therefore x = -2$ and $x = 1$ (twice).			

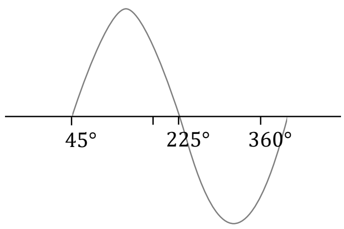
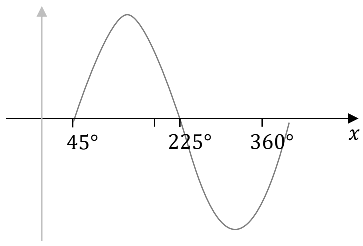
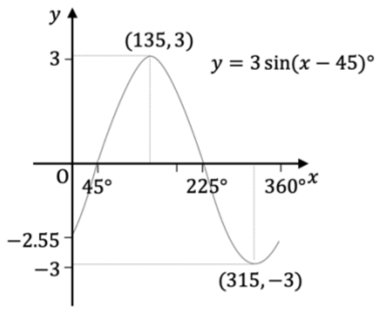
Section	Topic	Skills			
7.3	Finding a remainder when dividing a polynomial	To find a remainder when dividing a polynomial, use substitution, synthetic division or algebraic long division. **Example:** Find the remainder when $x^3 - 7x^2 - 4x + 26$ is divided by $(x + 2)$. $(-2)^3 - 7(-2)^2 - 4(-2) + 26 = -8 - 28 + 8 + 26 = -2$ $\therefore$ the remainder when $x^3 - 7x^2 - 4x + 26$ is divided by $(x + 2)$ is -2.			
7.4	Finding unknown coefficients of a polynomial	Substitute roots into equation and solve simultaneously. **Example:** Find the values of p and q if $(x + 2)$ and $(x - 1)$ are factors of $f(x) = x^3 + 4x^2 + px + q$. $(x + 2)$ is a factor $\therefore x = -2$ is a root $(-2)^3 + 4(-2)^2 - 2p + q = 0$ $8 - 2p + q = 0$ $q = 2p - 8$ $(x - 1)$ is a factor $\therefore x = 1$ is a root $(1)^3 + 4(1)^2 + p + q = 0$ $5 + p + q = 0$ $q = -p - 5$ Solve two equations simultaneously $2p - 8 = -p - 5$ … … $\therefore p = 1, q = -6$ and $f(x) = x^3 + 4x^2 + x - 6$			
7.5	Finding the points of intersection of curves	To find the points of intersection of two curves: • Equate the curves. • Rearrange to equal zero. • Solve to find x-coordinates (using synthetic division if necessary). • Substitute values into the curve to find the y-coordinates.			
7.6	Finding the equation of a polynomial from a graph	The roots of a polynomial are the x-coordinates at which the graph of the function cuts the x-axis. At the roots, the equation of the function $f(x) = 0$. We can use the **roots** and the **y-intercept** of the function to determine the equation. • Substitute the roots, $x = a, x = b, x = c$ in the polynomial with equation $\boldsymbol{y = k(x - a)(x - b)(x - c)}$. • Substitute the y-intercept or other coordinate into the equation to find the value of k.			
8	**Trigonometric Functions**				
National 5 Skills					
8.1	Finding the equation of a trigonometric function from a graph	In functions of the form $y = \boldsymbol{a} \sin \boldsymbol{b}x \pm \boldsymbol{c}$ and $y = \boldsymbol{a} \cos \boldsymbol{b}x \pm \boldsymbol{c}$ the **amplitude** of the graph is the positive value of $\boldsymbol{a}$. The **period** of the graph is the length of one full wave. For cosine and sine graphs, the period can be derived by **dividing $360°$ by b**. The value of $\boldsymbol{c}$ translates the graph **vertically**. In functions of the form $y = a \sin(x + \boldsymbol{b})$ and $y = a \cos(x + \boldsymbol{b})$, a is the amplitude and the value of b translates the graph **horizontally**.			
8.2	Solving trigonometric equations	Use the CAST diagram or graphical method to solve equations (see National 5 checklist).			

Section	Topic	Skills			
Higher Skills					
8.3	Exact values	It is important to know the exact values from the triangles below or the table.   We should also know the values of sine and cosine of $90°$, $180°$, $270°$ and $360°$.			
8.4	Solving trigonometric equations with exact values	Exact values can be used to solve trigonometric equations without a calculator.			
8.5	Solving trigonometric equations with radians	Trigonometric equations are usually solved using radians. The values in the table should be known. For solutions in radians, the ability to add fractions is desirable. **Example:** $2\cos x - \sqrt{3} = 0$ $2\cos x = \sqrt{3}$ $\cos x = \frac{\sqrt{3}}{2}$ $x = \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ $x = \frac{\pi}{6}, \frac{12\pi}{6} - \frac{\pi}{6}$ $x = \frac{\pi}{6}, \frac{11\pi}{6}$			

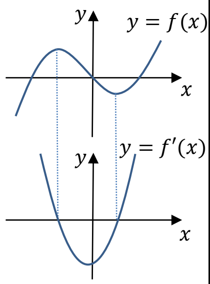
Degrees	Radians
360°	2π
180°	π
90°	$\frac{\pi}{2}$
60°	$\frac{\pi}{3}$
45°	$\frac{\pi}{4}$
30°	$\frac{\pi}{6}$
270°	$\frac{3\pi}{2}$

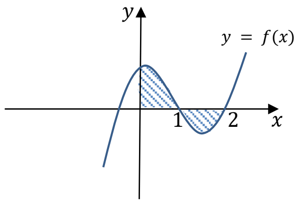
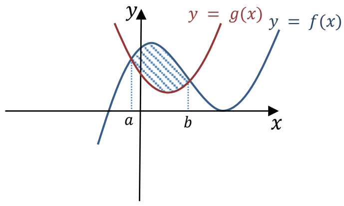
Section	Topic	Skills			
8.6	Points of intersection between a trigonometric graph and a line	To find the points of intersection of a trigonometric graph and a line: • Equate the equation of the line and the curve. • Solve to find x-coordinates. • Take the y-coordinate from the equation of the line.			
9	**Addition Formluae**				
9.1	Expanding the addition formulae	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$			
9.2	Using the addition formulae	**Example:** Given $\sin A = \frac{3}{5}$ and $\cos B = \frac{12}{13}$, show that $\sin(A + B) = \frac{56}{65}$ **Soln.** Use SOHCAHTOA to sketch triangles from the info given and use Pythagoras to find unknowns   Using the addition formulae: $\sin(A + B) = \sin A\cos B + \cos A\sin B$ $= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13} = \frac{56}{65}$			

Section	Topic	Skills			
9.3	Using the double angle formulae	The **double angle formulae** are the result of using the addition formulae on a trigonometric function with a repeated angle, **i.e.** $\sin(a+a) = \sin 2a$ and $\cos(a+a) = \cos 2a$. $\mathbf{\sin 2a = 2\sin a\cos a}$ $\mathbf{\cos 2a = \cos^2 a - \sin^2 a}$ $\mathbf{= 2\cos^2 a - 1}$ $\mathbf{= 1 - 2\sin^2 a}$ **Example:** Given $\sin x = \frac{3}{4}$, find the exact value of $\sin 2x$. $\sin x = \frac{3}{4}$ $\sqrt{(4)^2 - 3^2} = \sqrt{7}$  $\sin x = \frac{3}{4}, \cos x = \frac{\sqrt{7}}{4}$ $\sin 2x = 2\sin x\cos x = 2 \times \frac{3}{4} \times \frac{\sqrt{7}}{4} = \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}$			
9.4	Using trigonometric identities	**Example:** Show that $3\cos^2 x - \sin^2 x - 1 = 2\cos 2x$. Start with the left-hand-side and work towards the right. $LHS = 3\cos^2 x - \sin^2 x - 1$ $= 3\cos^2 x - (1 - \cos^2 x) - 1$ $= 4\cos^2 x - 2$ $= 2(2\cos^2 x - 1)$ $= 2\cos 2x$ $= RHS.$			
9.5	Solving equations using the double angle formulae	We can use the double angle formulae to solve trigonometric equations. To do so, we substitute the double angle function and then factorise the equation. **Example:** Solve the equation $\sin 2x° - \cos x° = 0$ for $0 \leq x \leq 360$. $\sin 2x° - \cos x° = 0$ $2\sin x°\cos x° - \cos x° = 0$ $\cos x°(2\sin x° - 1) = 0$ $\mathbf{\cos x° = 0}$ or $\mathbf{2\sin x° - 1 = 0}$ $\mathbf{x° = 90°, 270°}$ or $\mathbf{\sin x° = \frac{1}{2}}$ $\mathbf{x° = 30°, 150°}$ $x° = 30°, 90°, 150°, 270°$			
10	**The Wave Function**				
10.1	Using the wave function	**Step 1:** Expand using the addition formula, for example: $k\cos(x-\alpha) = k(\cos x\cos\alpha + \sin x\sin\alpha)$ $= \mathbf{k}\cos x\,\mathbf{\cos\alpha} + \mathbf{k}\sin x\,\mathbf{\sin\alpha} = \mathbf{k\cos\alpha}\cos x + \mathbf{k\sin\alpha}\sin x$			

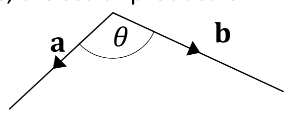
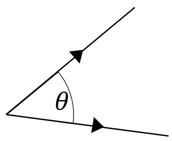
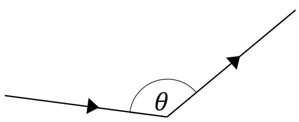
Section	Topic	Skills			
		Step 2: Equate the coefficients of a $\cos x +$ b $\sin x$ with the correct trigonometric function, being careful to ensure the correct coefficient. **e.g.** $k \cos \alpha =$ a and $k \sin \alpha =$ b **Step 3:** To calculate k, square and sum the coefficients, then square root the answer. This will eliminate $\sin \alpha$ and $\cos \alpha$. **Step 4:** To calculate α, use $\sin \alpha$ and $\cos \alpha$ to find $\tan \alpha$. $$\tan \alpha = \frac{k \sin \alpha}{k \cos \alpha} = \frac{\sin \alpha}{\cos \alpha}$$ Now use trigonometry to determine the quadrant. A good idea is to draw a CAST diagram and tick which quadrants are positive or negative (see worked examples).			
10.2	Solving trigonometric equations with the wave function	To solve trigonometric equations using the wave function: • Express in form $k \cos(x \pm \alpha)$ or $k \sin(x \pm \alpha)$. • Solve.			
10.3	Sketching the graph of $y = k \sin(x \pm \alpha)$ or $y = k \cos(x \pm \alpha)$	**Example:** Sketch and annotate the graph of $y = 3\sin(x - 45)°$ in the interval $0 \leq x \leq 360$. **Step 1:** Draw the wave on an x-axis and mark the roots. **Step 2:** Draw a y-axis.  The graph moves **right** by 45°.  **Step 3:** Fill in the rest of the information and add and remove the sections of the wave to fit the required range. 			
10.4	Maximum and minimum values	The maximum or minimum value of a trigonometric function of the form $y = k \sin(x \pm \alpha)$ or $y = k \cos(x \pm \alpha)$ within a given range can simply be found by determining the value of k. To find the x-coordinate where this value occurs, equate the function to the maximum or minimum value, then solve.			
11	**Calculus 1 - Differentiation**				
National 5 Skills					
11.1	Using indices	It is often necessary to use the laws of indices in differentiation. **Example:** $\dfrac{5}{2\sqrt[3]{x^2}} = \dfrac{5}{2x^{\frac{2}{3}}} = \dfrac{5}{2}x^{-\frac{2}{3}}$			

Section	Topic	Skills			
Higher Skills					
11.2	Differentiating functions	When differentiating a function, we multiply each term of the function by its power, then we take away 1 from the power. $\boldsymbol{f(x) = ax^n} \qquad \boldsymbol{f'(x) = nax^{n-1}}$			
11.3	Differentiation involving preparation of the function	When preparing a function for differentiation: • Break the function down into individual terms. • Express each term on the numerator of any fraction and in index form (**section 11.1**).			
11.4	Finding the rate of change of a function	When differentiating functions, the derivative of the function is the **rate of change** of the function, this rate of change is the **gradient of the tangent** to the curve of the function at any given point. To find the rate of change: • Differentiate. • Substitute. • Evaluate. Given the function $f(x) = 3x^3 + 3x^2 - 2x$, determine the rate of change of the function when $x = 3$. $f(x) = x^3 + 3x^2 - 2x$ $f'(x) = 9x^2 + 6x - 2$ $f'(3) = 9(3)^2 + 6(3) - 2 = 81 + 18 - 2 = 97$			
11.5	Finding the equation of a tangent	To find the equation of a tangent: • Substitute x into equation of curve to find y-coordinate. • Differentiate and substitute x value to calculate gradient. • Substitute into the equation of a line.			
11.6	Increasing or decreasing functions	To determine where a curve is increasing or decreasing: • Differentiate the function. • Determine where gradient is positive or negative from the derivate or a sketch of the derivative. **NB:** A function is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$.			
11.7	Finding the stationary points and their nature	To find the coordinates of the stationary points of a curve: • Differentiate the function. • Equate to zero. • Solve to find x-coordinate. • Substitute into original curve to y-coordinate. • Draw a nature table to determine their nature. • Answer the question. **Nature table:** x : SP^- , SP , SP^+ $\frac{dy}{dx}$: − , 0 , + Slope : \ , — , /			
11.8	Sketching the graph of a function	• Find stationary points and nature (**see 11.7**). • Find roots by solving function (when $y = 0$). • Find y-intercept (when $x = 0$). • Find large positive and large negative x. **e.g.** as $\boldsymbol{f(x) \to -\infty, x \to -\infty}$ and as $\boldsymbol{f(x) \to +\infty, x \to +\infty}$ • Sketch information on graph.			

Section	Topic	Skills			
11.9	Optimisation	**Optimisation** is the process of finding an **optimal solution** to a given problem. To find an optimal solution, we find the stationary points of the given function. In many cases, optimisation questions involve proving a formula from the given information. This typically involves using common formulae to express one of the variables in terms of the other, and then substituting the replaced variable. **NB:** Since the answer to part (a) is usually given, it is often possible to answer part (b) without doing part (a).			
11.10	Closed intervals	When determining the maximum and minimum values of a function within a closed interval • Find the maximum and minimum value in a closed interval. • Find the stationary points and determine their nature (**see 11.7**). • Find the y-coordinates at the extents of the interval. • Examine to see where the maximum and minimum values are.			
11.11	Sketching the derived function	• Sketch function. • Extend stationary points to the other coordinate axis. • Determine where the gradient, m, is +ve and −ve. **NB:** the gradient is +ve where the graph of $f'(x)$ is above the x-axis and −ve where it is below the x-axis. 			
12	**Calculus 2 – Integration**				
12.1	Basic Integration	When we integrate, we add one to the power and divide by the new power: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ When the integral is indefinite, **i.e.** it has no limits, we also add on the value C. This is known as the constant of integration.			
12.2	Integration involving preparation of the function	When preparing a function for integration: • Break the function down into individual terms. • Express each term on the numerator of any fraction and in index. form (see **section 11.1**).			
12.3	Definite integrals	To evaluate a definite integral: • Integrate function. • Evaluate between two limits. **Example:** Evaluate $\int_{-1}^{2} x^2 \, dx$. $\int_{-1}^{2} x^2 \, dx = \left[\frac{x^3}{3}\right]_{-1}^{2} = \left(\frac{(2)^3}{3}\right) - \left(\frac{(-1)^3}{3}\right) = \left(\frac{8}{3}\right) - \left(\frac{-1}{3}\right) = \frac{9}{3} = 3$			
12.4	Area under curves	• Find the area above the x-axis • Find the area below the x-axis (ignore the negative) • Add them together			

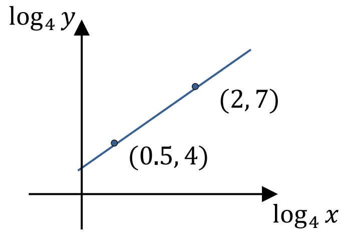
Section	Topic	Skills			
		Example: $\int_0^1 f(x)\,dx$ and $\int_1^2 f(x)\,dx$ **NB:** The area below the x-axis will give a negative answer. The area should be written as a positive value. 			
12.5	Area between two curves	To evaluate a definite integral: • Set the curves equal to each other and solve to find the limits. • Set up integral with: $\int [upper\ curve - lower\ curve]\ dx$ $\int_a^b [f(x) - g(x)]\ dx$ • Evaluate answer. 			
12.6	Differential equations	Equations of the form $\frac{dy}{dx} = ax + b$ are called differential equations. They are solved by integration **Example:** The curve $y = f(x)$ is such that $\frac{dy}{dx} = 9x^2$. The curve passes through $(1, 5)$. Express y in terms of x. $y = \int 9x^2\,dx = 3x^3 + C$ at $(1, 5)$, $5 = 3(1)3 + C$ $5 = 3 + C$ $C = 2 \quad \therefore \quad y = 3x^3 + 2$			
13	**Calculus 3 – Further Calculus**				
13.1	Differentiating composite functions – the chain rule	When we differentiate **composite functions** (**see section 4.3**), we can use the **Chain Rule**. With this rule, we differentiate the **outer** function and multiply by the derivative of the **inner** function: $$f(x) = (ax + b)^n$$ $$f'(x) = n(ax + b)^{n-1} \times a$$ When identifying the **outer** and **inner** function, consider where the brackets would be that surround the inner function.			
13.2	Differentiating trigonometric functions	It is important to identify the **inner** and **outer** function, which is not as obvious as algebraic questions. It is important to recognise where the brackets would be, **e.g.** $$\frac{d}{dx}\sin ax = a\cos(ax) \qquad \frac{d}{dx}\cos ax = -a\sin(ax)$$			
13.3	Integrating composite functions	To integrate **composite functions**: • **Integrate** the outer function. • Divide by the **derivative** of the inner function. $$\int (ax + b)^n\,dx$$ $$= \frac{(ax + b)^{n+1}}{(n + 1) \times a} + C$$			

Section	Topic	Skills			
13.4	Integrating trigonometric functions	When we integrate **trigonometric functions**, we have the following: $\int \cos ax\,dx = \frac{\sin ax}{a} + C$ $\int \sin ax\,dx = -\frac{\cos ax}{a} + C$			
14	**Vectors**				
National 5 Skills					
14.1	Adding and subtracting vectors in component form	Add and Subtract 2D and 3D vector components: $\mathbf{a} = \begin{pmatrix}1\\1\\4\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}3\\2\\5\end{pmatrix}$ $\mathbf{a}+\mathbf{b} = \begin{pmatrix}1+3\\1+2\\4+5\end{pmatrix}$			
14.2	Using position vectors	In mathematics, a **position vector** is a vector from the origin to a point on a coordinate axis. **Example:** The position vector of coordinate $\mathrm{A}(2,4,5)$ is $\mathbf{a} = \begin{pmatrix}2\\4\\5\end{pmatrix}$. If we know the coordinates of points A and B, then $\overrightarrow{\mathrm{AB}} = \mathbf{b} - \mathbf{a}$.			
14.3	Magnitude of vectors	To calculate the magnitude of a vector, we use the following formula: $\lvert\mathbf{u}\rvert = \sqrt{x^2+y^2}$ or in three dimensions: $\lvert\mathbf{u}\rvert = \sqrt{x^2+y^2+z^2}$			
Higher Skills					
14.4	Finding unit vectors	A **unit vector** is a vector with a magnitude of 1 unit. Example: Find the unit vector $\mathbf{u}$ parallel to vector $\mathbf{a} = \begin{pmatrix}-8\\-1\\4\end{pmatrix}$ $\lvert\mathbf{a}\rvert = \sqrt{(-8)^2+(-1)^2+4^2} = 9$ $\therefore \mathbf{u} = \frac{1}{9}\begin{pmatrix}-8\\-1\\4\end{pmatrix} = \begin{pmatrix}-{}^{8}/_{9}\\-{}^{1}/_{9}\\{}^{4}/_{9}\end{pmatrix}$			
14.5	Working with unit vectors	For example, for vector $\mathbf{v}$, we can express the vector in either component form or unit vector form: **Example:** $\mathbf{v} = \begin{pmatrix}4\\3\\-7\end{pmatrix} = 4\mathbf{i}+3\mathbf{j}-7\mathbf{k}$.			
14.6	Collinearity	Points are said to be collinear if they line on the same line. To show points are collinear using vectors; show (a) they are parallel by demonstrating one vector is a scalar multiple of the other and (b) that they share a common point. **Example:** Show that $\mathrm{A}(-3,4,7)$, $\mathrm{B}(-1,8,3)$ and $\mathrm{C}(0,10,1)$ are collinear. $\overrightarrow{\mathrm{AB}} = \mathbf{b}-\mathbf{a} = \begin{pmatrix}-1\\8\\3\end{pmatrix} - \begin{pmatrix}-3\\4\\7\end{pmatrix} = \begin{pmatrix}2\\4\\-4\end{pmatrix} = 2\begin{pmatrix}1\\2\\-2\end{pmatrix}$ $\overrightarrow{\mathrm{BC}} = \mathbf{c}-\mathbf{b} = \begin{pmatrix}0\\10\\1\end{pmatrix} - \begin{pmatrix}-1\\8\\3\end{pmatrix} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$ $\overrightarrow{\mathrm{AB}} = 2\overrightarrow{\mathrm{BC}}$ and point B is common $\therefore$ A, B and C are collinear.			
14.7	Determining the coordinates of a division point on a line	**Example:** P is the point $(6,3,9)$ and R is $(12,6,0)$. Find the coordinates of Q, such that Q divides PR in the ratio 2:1			

Section	Topic	Skills			
		$\frac{\overrightarrow{PQ}}{\overrightarrow{QR}} = \frac{2}{1}$ $\overrightarrow{PQ} = 2\overrightarrow{QR}$ $\boldsymbol{q} - \boldsymbol{p} = 2(\boldsymbol{r} - \boldsymbol{q})$ $\boldsymbol{q} - \boldsymbol{p} = 2\boldsymbol{r} - 2\boldsymbol{q}$ $3\boldsymbol{q} = 2\boldsymbol{r} + \boldsymbol{p}$ $3\boldsymbol{q} = 2\begin{pmatrix}12\\6\\0\end{pmatrix} + \begin{pmatrix}6\\3\\9\end{pmatrix} = \begin{pmatrix}30\\15\\9\end{pmatrix}$ $\boldsymbol{q} = \begin{pmatrix}10\\5\\3\end{pmatrix} \quad \therefore \mathrm{Q}\,(10, 5, 3).$ **NB:** This is an alternative method to the one used in the book.			
14.8	Determining the ratio of division of a line segment	**Example:** $\mathrm{A}(-2, -1, 4)$, $\mathrm{B}(1, 5, 7)$ and $\mathrm{C}(7, 17, 13)$ are collinear. Determine the ratio in which B divides AC. $\overrightarrow{AB} = \boldsymbol{b} - \boldsymbol{a} = \begin{pmatrix}1\\5\\7\end{pmatrix} - \begin{pmatrix}-2\\-1\\4\end{pmatrix} = \begin{pmatrix}3\\6\\3\end{pmatrix}$ $\overrightarrow{BC} = \boldsymbol{c} - \boldsymbol{b} = \begin{pmatrix}7\\17\\13\end{pmatrix} - \begin{pmatrix}1\\5\\7\end{pmatrix} = \begin{pmatrix}6\\12\\6\end{pmatrix} = 2\begin{pmatrix}3\\6\\3\end{pmatrix} = 2\overrightarrow{AB}$ $2\overrightarrow{AB} = \overrightarrow{BC}$ $\frac{\overrightarrow{AB}}{\overrightarrow{BC}} = \frac{1}{2} \quad \therefore \overrightarrow{AB} : \overrightarrow{BC}$ $1 : 2$ **NB:** This is an alternative method to the one used in the book.			
14.9	The scalar product	When given an angle between two vectors, the scalar product is calculated using $\mathbf{a}.\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta$  **NB:** To find the angle between the two vectors θ, the vectors must be pointing away from or towards each other and $0 \leq \theta \leq 180$. When given component form, i.e. if $\boldsymbol{a} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}$, the scalar product is calculated using $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$. **NB:** When vectors meet at 90°, the scalar product is zero.			
14.10	Finding the angle between two vectors	The angle between two vectors is calculated by rearranging the scalar product formula $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}$ which can be expanded to $\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\|\mathbf{a}\|\|\mathbf{b}\|}$. **NB:** to find the angle between two vectors, the vectors must be pointing away from or towards each other. They must not be going in the same direction. **e.g.**  Not 			
14.11	Properties of the scalar product	The **scalar product** has several properties that we need to know for Higher Mathematics: $\mathbf{a}\,.\,\mathbf{b} = \mathbf{b}\,.\,\mathbf{a}$ The commutative law applies to vectors.			

Section	Topic	Skills			
		$\mathbf{a}\,.\,(\mathbf{b}+\mathbf{c}) = \mathbf{a}\,.\,\mathbf{b}+\mathbf{a}\,.\,\mathbf{c}$ The distributive law also applies to vectors. $\mathbf{a}\,.\,\mathbf{a} = \lvert\mathbf{a}\rvert^2$ Equal vectors $\mathbf{a}\,.\,\mathbf{b} = \mathbf{0}$ Perpendicular vectors.			
14.12	Vector pathways	In Higher Mathematics, we also need to use **vector pathways** in **three dimensions**. As with two dimensional pathways, this is a description of the pathway from the beginning to the end of the vector. These vector pathways can also be described either in terms of their components or by combinations of known vectors.			
15	**Logarithmic & Exponential Functions**				
15.1	Using logarithms and exponentials	If we have an exponential function $\boldsymbol{y = a^x}$, we can write this in logarithmic form as $\boldsymbol{x = \log_a y}$. **Example:** $3 = \log_a 8$ $a^3 = 8$ $a = 2$			
15.2	Finding the equation of a function from its graph	To determine the equation of the graph of an exponential or logarithmic function, substitute the information from the graph into the equation.			
15.3	Sketching a graph from its equation	To sketch the graph of an **exponential function** from its equation, two coordinates are needed: the coordinate when $x = 0$ and the coordinate when $x = 1$. To sketch the graph of a **logarithmic function** from its equation, two coordinates are needed: the coordinate when $y = 0$ and the coordinate when $y = 1$.			
15.4	Sketching the inverse function	The graph of an inverse function is reflected in the line $y = x$. To sketch the graph of an inverse function: **Step 1:** Sketch the graph of the function. **Step 2:** Sketch the line $y = x$. **Step 3:** Sketch the inverse graph as a reflection of the original function; remember to annotate the image of each coordinate. **NB:** When sketching the inverse function, rotate the page so that the line $y = x$ is vertical, this will make reflecting the original easier.			
15.5	Sketching related graphs	The graph transformations of logarithmic and exponential functions follow the same principles as the functions we have already seen in **section 5**.			
15.6	Evaluating expressions using the laws of logarithms	There are five laws when using logarithms that are useful for evaluating expressions and solving equations: **The First Law:** $\boldsymbol{\log_a b + \log_a c = \log_a bc}$ $\log_4 8 + log_4 2 = \log_4(8 \times 2) = \log_4 16 = 2$ (since $4^2 = 16$)			

Section	Topic	Skills			
		The Second Law: $\log_a b - \log_a c = \log_a \frac{b}{c}$ $\log_4 8 - \log_4 2 = \log_4 \frac{8}{2} = \log_4 4 = 1$ (since $4^1 = 4$) **The Third Law:** $\log_a b^c = c\log_a b$ $\frac{1}{3}\log_9 27 = \log_9 27^{\frac{1}{3}} = \log_9 3 = \frac{1}{2}$ (since $9^{\frac{1}{2}} = 3$) **The Fourth Law:** $\log_a 1 = 0$ $\log_{12} 1 = 0$ (since $12^0 = 1$) **The Fifth Law:** $\log_a a = 1$ $\log_5 5 = 1$ (since $5^1 = 5$)			
15.7	Solving logarithmic and exponential equations	**Example 1:** Solve $\log_5(x+1) + \log_5(x-3) = 1$. $\log_5(x+1)(x-3) = 1$ (using first law) $(x+1)(x-3) = 5$ (since $5^1 = 5$) $x^2 - 2x - 3 = 5$ (solve for x) $x^2 - 2x - 8 = 0$ $(x+2)(x-4) = 0$ $\therefore x = -2,\ x = 4$ **Example 2:** Find x if $4\log_x 6 - 2\log_x 4 = 1$. $\log_x 6^4 - \log_x 4^2 = 1$ $\log_x \frac{6^4}{4^2} = 1$ $\frac{6^4}{4^2} = x$ $x = \frac{2^4 \times 3^4}{2^4}$ (since $6^4 = 2^4 \times 3^4$ and $4^2 = 16 = 2^4$) $x = 3^4$ $x = 81$			
15.8	Exponential growth and decay	When working with exponential growth and decay, we will primarily use the exponential function (e^x or $\exp x$) and the natural log function ($\ln x$ or $\log_e x$). • For finding an initial value; substitue given values in to equation to determine the initial value. • For finding a half-life, make the equation equal to one half. **Example:** In the function $A_t = A_0 e^{-0.004t}$, A_t represents micrograms of a radioactive substance remaining after time t and A_0 represents the initial value. **(a)** Calculate the initial value if there are 500 microgram after 100 years. **(b)** Calculate the half-life of the substance. **(a)** $A_t = A_0 e^{-0.004t}$ $500 = A_0 e^{-0.004 \times 100}$ $500 = 0.67 \ldots \times A_0$ $A_0 = 746$ micrograms (3 s.f.) **(b)** $373 = 746e^{-0.004t}$ $\frac{1}{2} = e^{-0.004t}$ $\ln\frac{1}{2} = \ln e^{-0.004t}$ $-0.004t = \ln\frac{1}{2}$ $\therefore t = 173$ years			

Section	Topic	Skills			
15.9	Log-linear & log-log graphs (experimental data)	In experimental data questions, two types of exponential functions are considered, $y = kx^n$ and $y = ab^x$. **Functions of the form $y = kx^n$** **Example 1:** Two variables x and y are related by the equation $y = kx^n$. When $\log y$ is plotted against $\log x$, a straight line passing through the points $(0, 2)$ and $(3, 11)$ is obtained. Find the values of k and n.  **Step 1:** Express $y = kx^n$ in logarithmic form. $\log_4 y = n \log_4 x + \log_4 k$ **Step 2:** Either use the same method as before, calculating m, or substitute the coordinates and solve simultaneously. $7 = 2m + \log_4 k$ **(A)** $4 = 0.5m + \log_4 k$ **(B)** **(A) − (B)** $3 = 1.5m$ $\therefore m = 2$ Substitute $m = 2$ into (A) $7 = 4 + \log_4 k$ $3 = \log_4 k$ $k = 4^3 = 64$ $\therefore k = 64, m = 2$ and $y = 64x^2$ **Functions of the form $y = ka^x$** **Example 2:** Two variables x and y are related by the equation, $y = ka^x$. When $\log y$ is plotted against x, a straight line passing through the points $(0, 2)$ and $(3, 11)$ is obtained. Find the values of k and a. **Step 1:** Express $y = ka^x$ in logarithmic form. $\log_3 y = x \log_3 a + \log_3 k$ **Step 2:** Determine k. $\log_3 k = 2 \therefore k = 9$ **Step 3:** Determine m. This can be done using the gradient formula with the two coordinates or by substitution of $\log_3 k$ and the other coordinate. $11 = 3\log_3 a + 2$ $9 = 3\log_3 a$ $3 = \log_3 a$ $a = 27$ $\therefore k = 9, a = 27$ and $y = 27(3)^x$			